Math 124E - Summer 2020 - Exam #2
DUE TUESDAY, JUNE 30, 2020 BY 11:59PM

Name: Answer Key

HONOR CODE: On my honor, I have neither given nor received any aid on this examination that is not explicitly allowed in the instructions.

Signature: ______________________________

Instructions: You may review the videos in MyMathLab or the course website, use the e-book, the MyMathLab homework and quizzes, and your calculator when working on this exam. **You may not receive help from anyone else, give help to anyone else, discuss any aspect of the exam or any items related to the exam with anyone else, or use any resources not specified in the previous sentence.** You may submit your answers and scratch work either on a printed copy of this exam or on your own paper. If you use your own paper, you do **not** need to copy the question; just be sure you clearly label which question the scratch work and answer belong to. If I can't tell with certainty which question any scratch work or answer belongs to, you will not receive credit for that work or answer. If you use your own paper, you **DO** need to copy the honor code above and sign it. To submit your scratch work and answers, you can either scan your work (if you have access to a scanner) or take pictures with your cell phone, then email me your scans or pictures. Be sure the writing in your scans or pictures is dark enough and clear enough that I can easily read what you've written. If I can't read what you've written, I can't give you credit for it. Make sure your final answers are clearly labeled. **SHOW ALL WORK ON THIS EXAM IN ORDER TO RECEIVE FULL CREDIT!!!**

No.	Score
1	/4
2	/7
3	/4
4	/4
5	/14
6	/6
7	/6
8	/6

No.	Score
9	/5
10	/12
11	/8
12	/5
13	/11
14	/8
Total	/100

1. Determine whether the following equation defines y as a function of x. **Explain your answer!** (4 points)

$$x = |y| + \cancel{1}$$

$$-1 \qquad -\cancel{1}$$

$$|y| = x - 1$$

$$y = \pm(x-1)$$

Not a function because one x-value can have two different y-values

2. Evaluate the function $f(x) = 2x^2 - 4$ at the given values of the independent variable and simplify.

(a) $f(-4)$ (2 points)

$$f(-4) = 2(-4)^2 - 4$$
$$= 2(16) - 4$$
$$= 32 - 4 = 28$$

(b) $f(a)$ (2 points)

$$f(a) = 2a^2 - 4$$

(c) $f(x+1)$ (3 points)

$$f(x+1) = 2\underbrace{(x+1)^2}_{} - 4$$

$$(x+1)(x+1) = x^2 + x + x + 1 = x^2 + 2x + 1$$

$$= 2(x^2 + 2x + 1) - 4$$

$$= 2x^2 + 4x + 2 - 4$$

$$= 2x^2 + 4x - 2$$

3. Without graphing, determine whether the function is even, odd or neither. Then determine whether the function's graph is symmetric with respect to the x-axis, y-axis, the origin, or none of these. **Explain your answer!** (4 points)

$$\cancel{x^2} + y = -3$$
$$\cancel{-x^2} \qquad -x^2$$

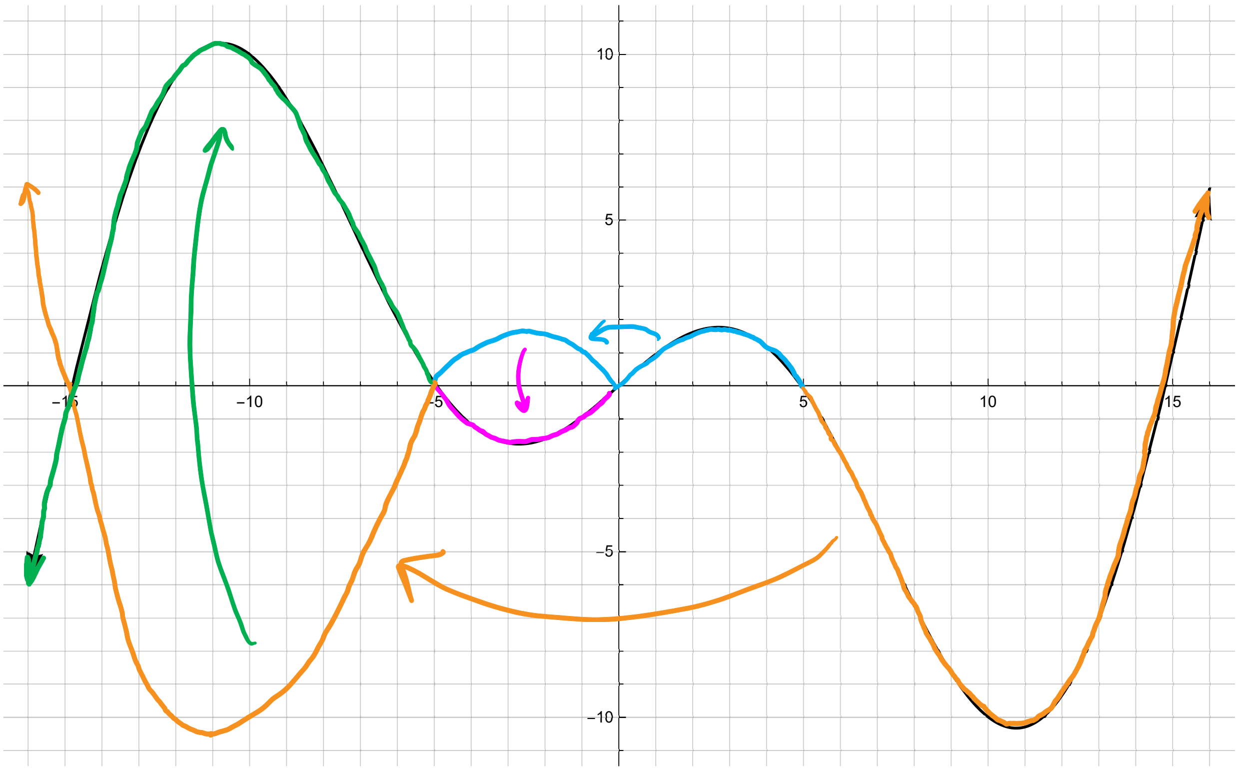
$$y = -x^2 - 3$$

$$f(x) = -x^2 - 3$$
$$f(-x) = -(-x)^2 - 3$$
$$= -x^2 - 3$$
$$= f(x)$$

$f(-x) = f(x) \Rightarrow$ even function

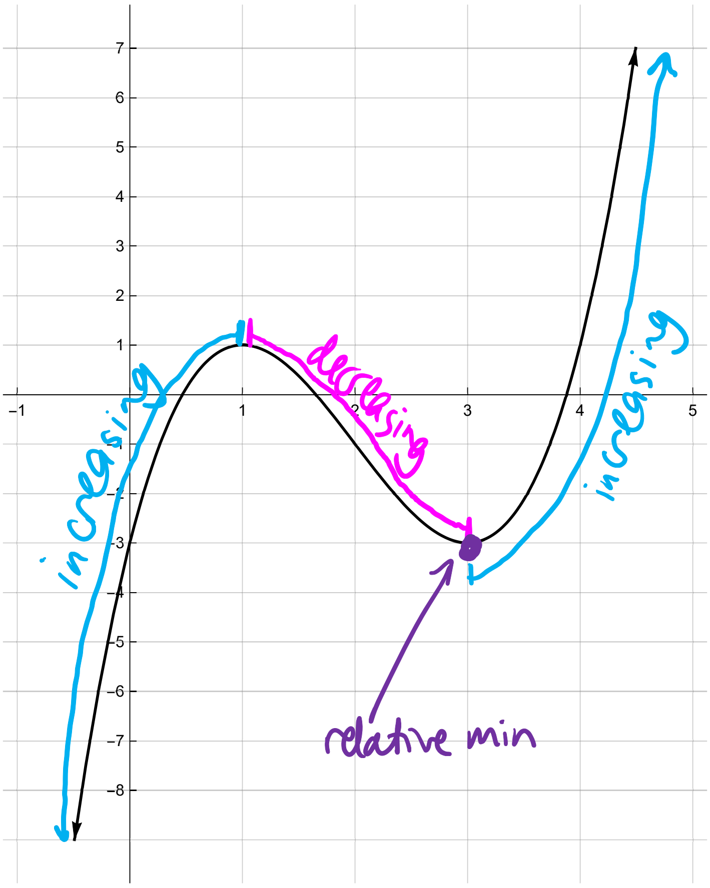
$\Rightarrow$ symmetric about y-axis

4. Use possible symmetry of the graph to determine whether it is the graph of an even function, an odd function, or a function that is neither even or odd. **Explain your answer!** (4 points)



If you take the right side of the graph and reflect it across the y-axis, then reflect that across the x-axis, you get the left side of the graph; in other words, this graph is symmetric about the origin because $f(-x) = -f(x)$. Therefore, this is the graph of an odd function.

5. Use the graph to find the following.



(a) The intervals on which f is increasing. (3 points)

$(-\infty, 1) \cup (3, \infty)$

(b) The intervals on which f is decreasing. (3 points)

$(1,3)$

(c) The number at which f has a relative minimum. (2 points)

$x = 3$

(d) The relative mimimum of f. (2 points)

$y = -3$

(e) $f(0)$ (2 points)

$f(0) = -3$

(f) The values of x for which $f(x) = 1$. (2 points)

$x = 1, 4$

6. Use the piecewise function to answer the following.

$$f(x) = \begin{cases} 2x - 1 & \text{if } x \leq 2 \\ -4 & \text{if } x > 2 \end{cases}$$

(a) Find the value of $f(-3)$. (2 points)

Use top expression, since $-3 \leq 2$

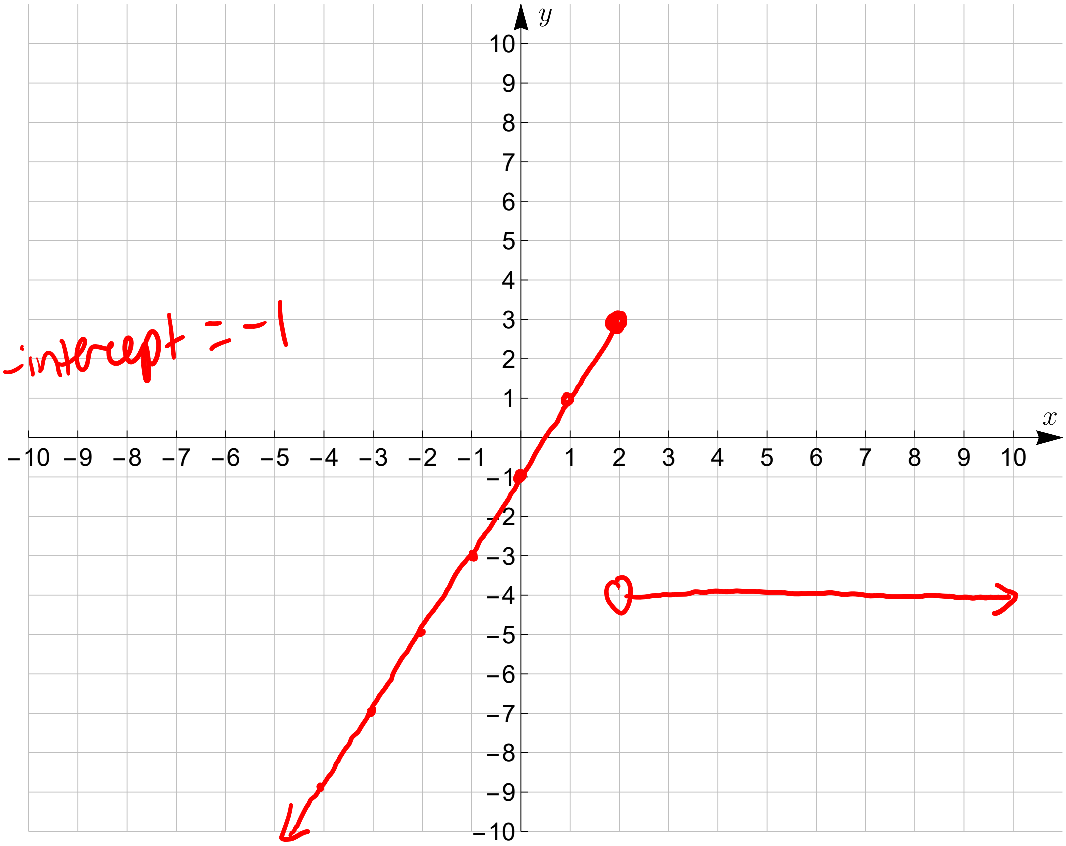
$f(-3) = 2(-3) - 1 = -6 - 1 = -7$

(b) Graph the function. (4 points)

$y = 2x - 1$

y-intercept $= -1$

slope $= \frac{2}{1}$



7. Find an simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$, $h \neq 0$ for the given function. (6 points)

$$f(x) = 3x^2 - 1$$

$$f(x+h) = 3\underbrace{(x+h)^2}_{} - 1 =$$

$$(x+h)(x+h) = x^2 + xh + xh + h^2$$
$$= x^2 + 2xh + h^2$$

$$= 3(x^2+2xh+h^2) - 1 = 3x^2 + 6xh + 3h^2 - 1$$

$$\frac{f(x+h)-f(x)}{h} = \frac{(3x^2+6xh+3h^2-1)-(3x^2-1)}{h}$$

$$= \frac{\cancel{3x^2}+6xh+3h^2\cancel{-1}\cancel{-3x^2}\cancel{+1}}{h}$$

$$= \frac{6xh+3h^2}{h}$$

$$= \frac{\cancel{h}(6x+3h)}{\cancel{h}}$$

$$= 6x+3h$$

8. Use the given conditions to write an equation for the line in slope-intercept form. (6 points)

Passing through $(-1, 2)$ and $(-6, -4)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-2}{-6-(-1)} = \frac{-4-2}{-6+1} = \frac{-6}{-5} = \frac{6}{5}$$

Option #1: $y - y_1 = m(x - x_1)$

$$y - 2 = \frac{6}{5}(x - (-1))$$

$$y - 2 = \frac{6}{5}(x + 1)$$

$$y - \cancel{2} = \frac{6}{5}x + \frac{6}{5}$$

$$+\cancel{2} \qquad\qquad +2$$

$$y = \frac{6}{5}x + \frac{16}{5}$$

$$\frac{6}{5} + 2$$

$$\frac{6}{5} + \frac{2}{1}$$

$$\frac{6}{5} + \frac{10}{5}$$

$$\frac{16}{5}$$

Option #2: $y = mx + b \Rightarrow y = \frac{6}{5}x + b$

$$2 = \frac{6}{5}(-1) + b$$

$$2 = -\cancel{\frac{6}{5}} + b$$

$$+\frac{6}{5} \qquad +\cancel{\frac{6}{5}}$$

$$b = \frac{16}{5} \Rightarrow y = \frac{6}{5}x + \frac{16}{5}$$

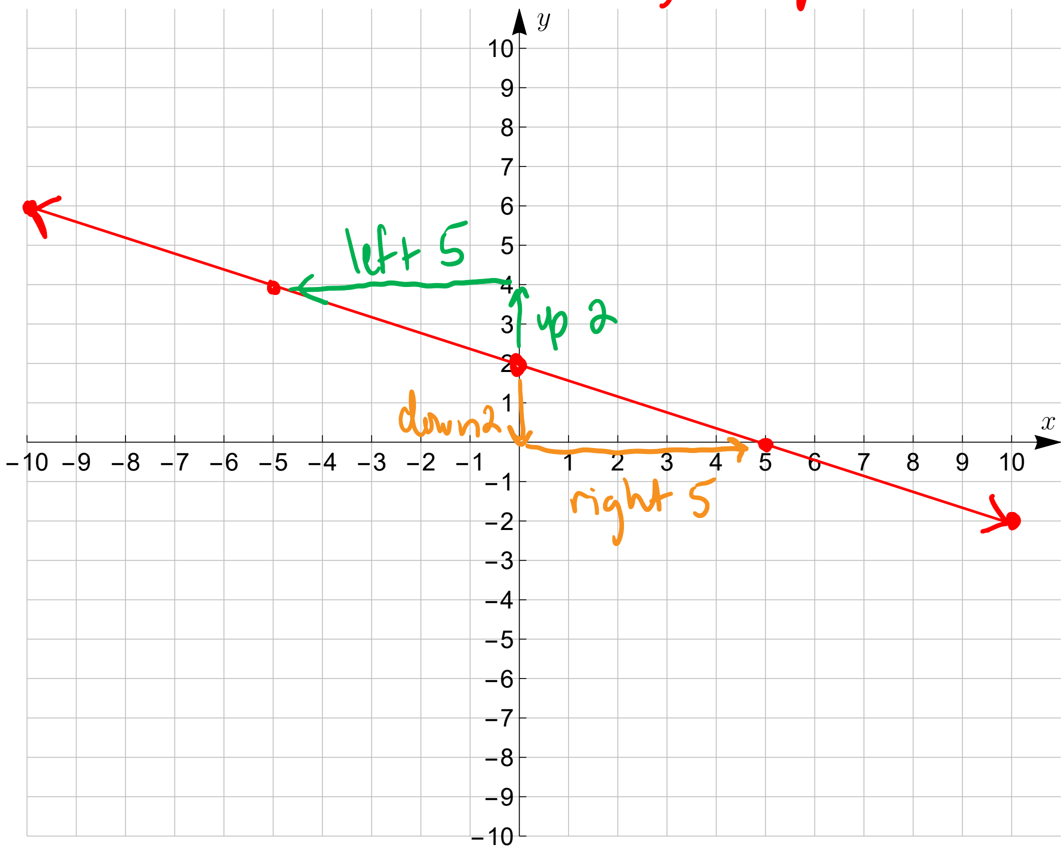
Note: We could have used $(-6, -4)$ instead of $(-1, 2)$ for either option

9. Give the slope and y-intercept of the line whose equation is given. Then graph the linear function. (5 points)

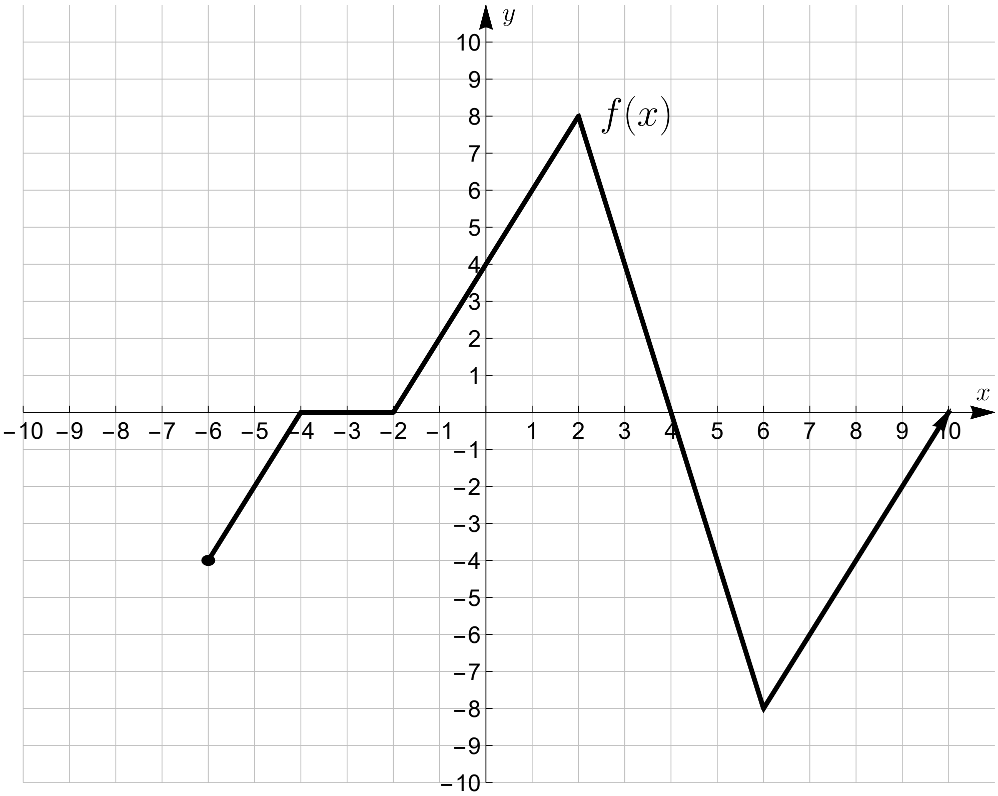
$$y = -\frac{2}{5}x + 2$$

slope = $\frac{\text{down } 2}{\text{right } 5}$ or $\frac{\text{up } 2}{\text{left } 5}$

y-intercept



10. Use the graph of $y = f(x)$ to graph the following functions. (6 points each)

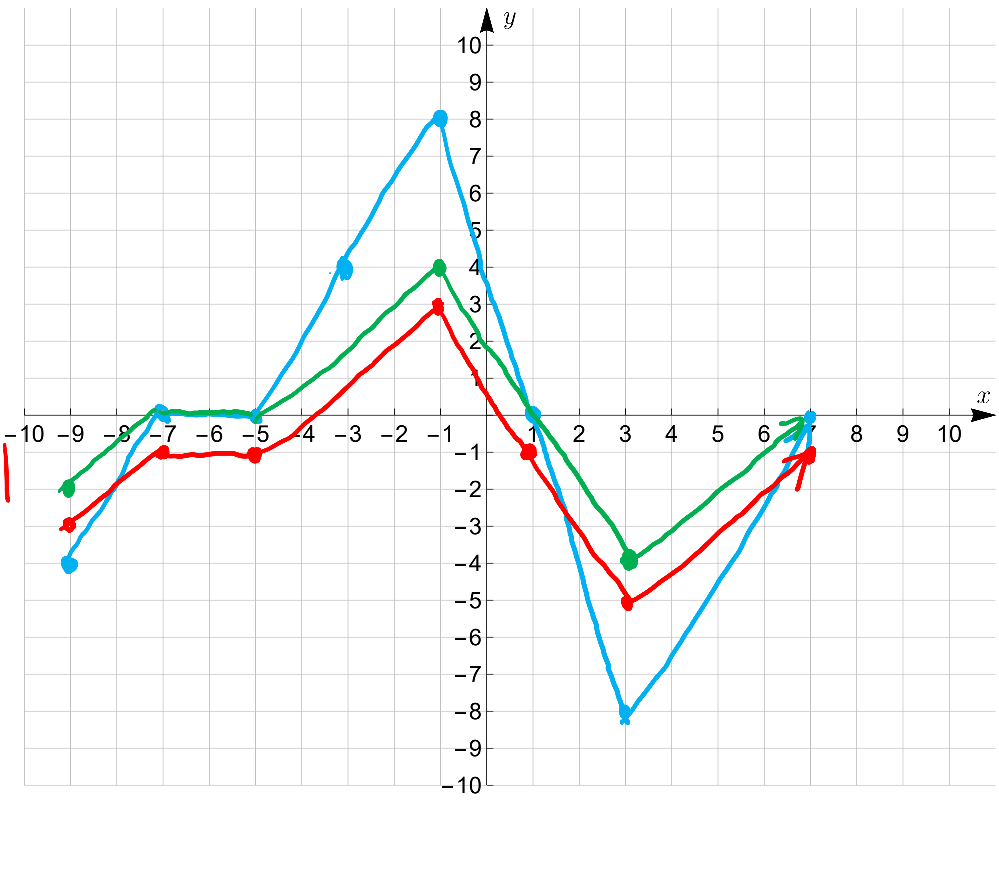


(a) $g(x) = \frac{1}{2}f(x+3) - 1$

1st: left by 3

2nd: vertical shrink by $\frac{1}{2}$

3rd: down by 1

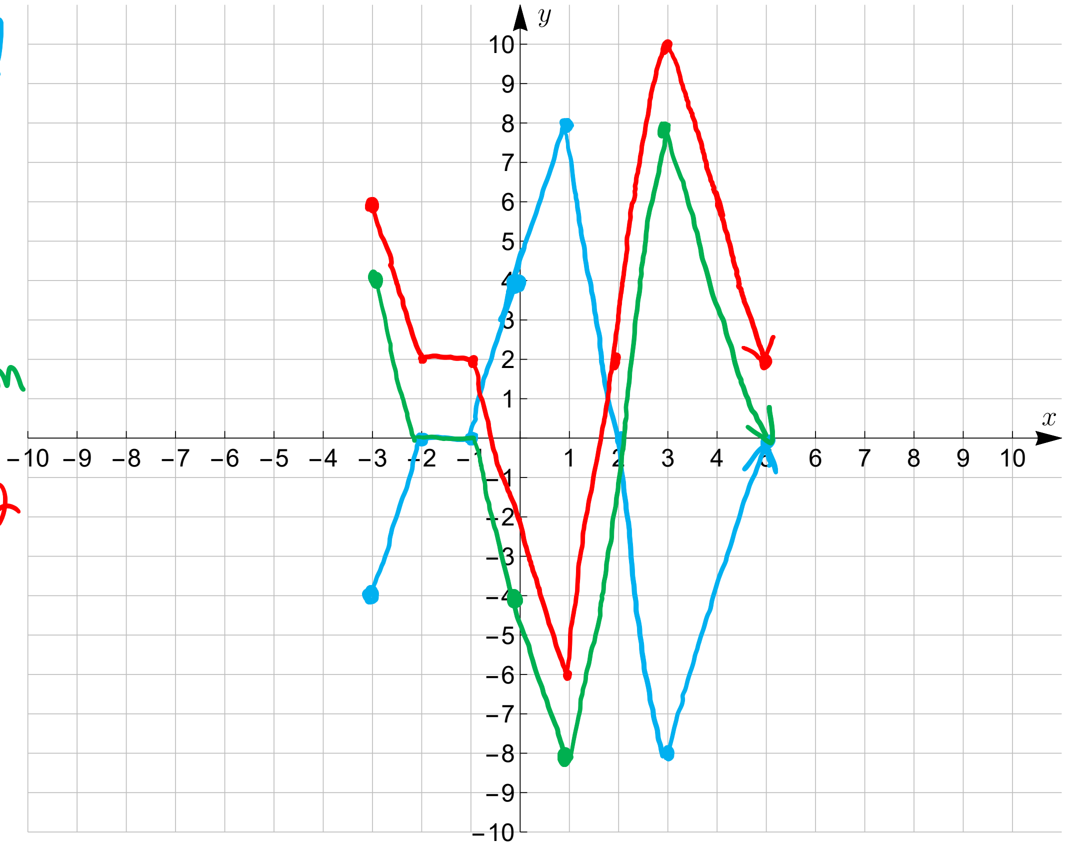


(b) $h(x) = -f(2x) + 2$

1st: horizontal shrink by $\frac{1}{2}$

2nd: vertical reflection

3rd: up by 2



11. For $f(x) = 3x^2 - 2$ and $g(x) = 4x + 1$, find the following functions. (4 points each)

(a) $(f \circ g)(2)$

Option #1: $g(2) = 4(2)+1 = 8+1 = 9$

$f(g(2)) = f(9) = 3(9)^2 - 2 = 3(81) - 2$

$= 243 - 2 = 241$

Option #2: $f(g(x)) = f(4x+1) = 3\underbrace{(4x+1)^2}_{(4x+1)(4x+1) = 16x^2+8x+1} - 2$

$= 3(16x^2 + 8x + 1) - 2$

$= 48x^2 + 24x + 3 - 2$

$= 48x^2 + 24x + 1$

$f(g(2)) = 48(2)^2 + 24(2) + 1 = 48(4) + 24(2) + 1$

$= 192 + 48 + 1 = 241$

(b) $(g \circ f)(x)$

$g(f(x)) = g(3x^2 - 2) = 4(3x^2 - 2) + 1$

$= 12x^2 - 8 + 1$

$= 12x^2 - 7$

12. The function $f(x) = \sqrt{2x+4} - 5$ is one-to-one. Find an equation for $f^{-1}(x)$. (5 points)

$$y = \sqrt{2x+4} - 5$$

$$x = \sqrt{2y+4} - \cancel{5}$$

$$+5 \qquad\qquad +\cancel{5}$$

$$(x+5)^2 = (\cancel{\sqrt{2y+4}})^{\cancel{2}}$$

$$(x+5)^2 = 2y + \cancel{4}$$

$$-4 \qquad\qquad \cancel{-4}$$

$$\frac{(x+5)^2 - 4}{2} = \frac{\cancel{2}y}{\cancel{2}}$$

$$f^{-1}(x) = \frac{(x+5)^2 - 4}{2} = \frac{x^2 + 10x + 25 - 4}{2}$$

$$= \frac{x^2 + 10x + 21}{2}$$

13. Use the function to answer the following.

$$f(x) = 2x^2 - 24x + 64$$

(a) Without graphing, find the coordinates of the vertex of the function. (3 points)

$x = -\frac{b}{2a} = -\frac{-24}{2(2)} = \frac{24}{4} = 6$

$f(6) = 2(6)^2 - 24(6) + 64 = 2(36) - 24(6) + 64 = 72 - 144 + 64$
$= -8$

vertex = $(6, -8)$

(b) Write the function in vertex form. (3 points)

$f(x) = 2(x-6)^2 - 8$

(c) Graph the function. (5 points)

x-intercepts:

$0 = 2(x-6)^2 - \cancel{8}$
$+8 \qquad +\cancel{6}$

$\frac{8}{2} = \frac{\cancel{2}(x-6)^2}{\cancel{2}}$

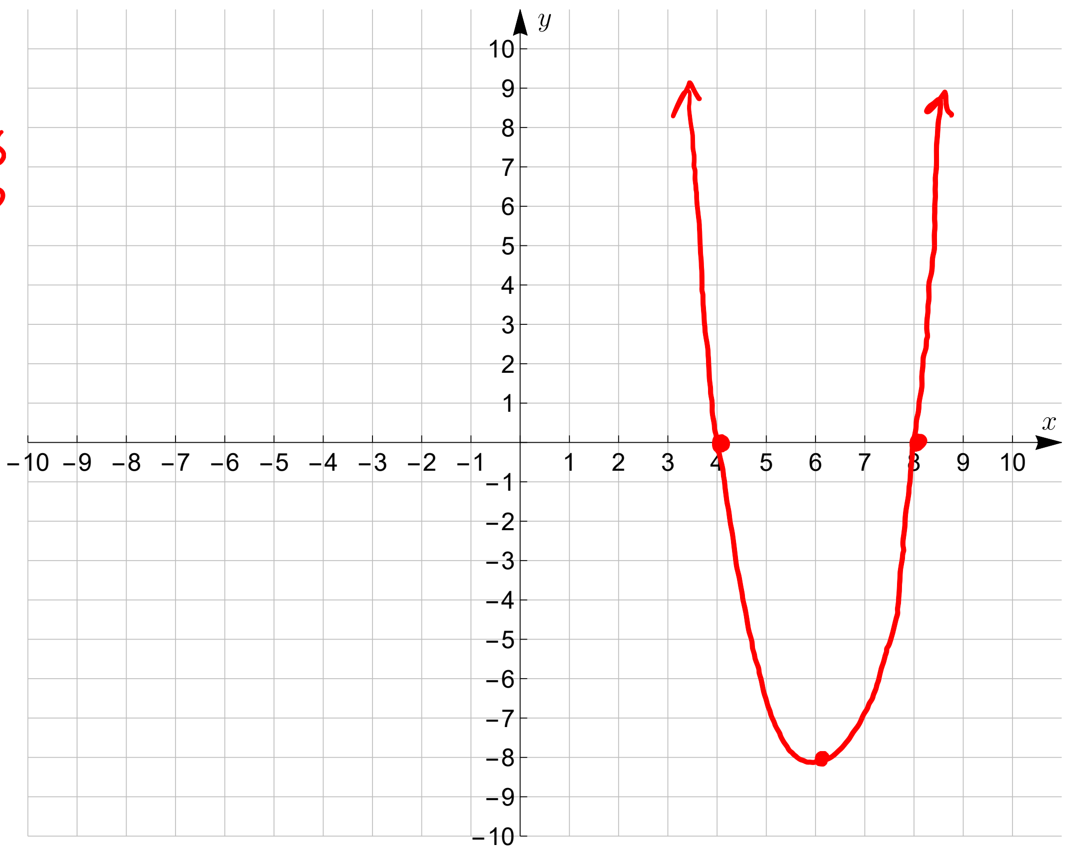
$\pm\sqrt{4} = \sqrt{(x-6)^2}$

$\pm 2 = x - \cancel{6}$
$+6 \qquad +6$

$x = 6 \pm 2$

$x = 6 + 2 = 8$

$x = 6 - 2 = 4$

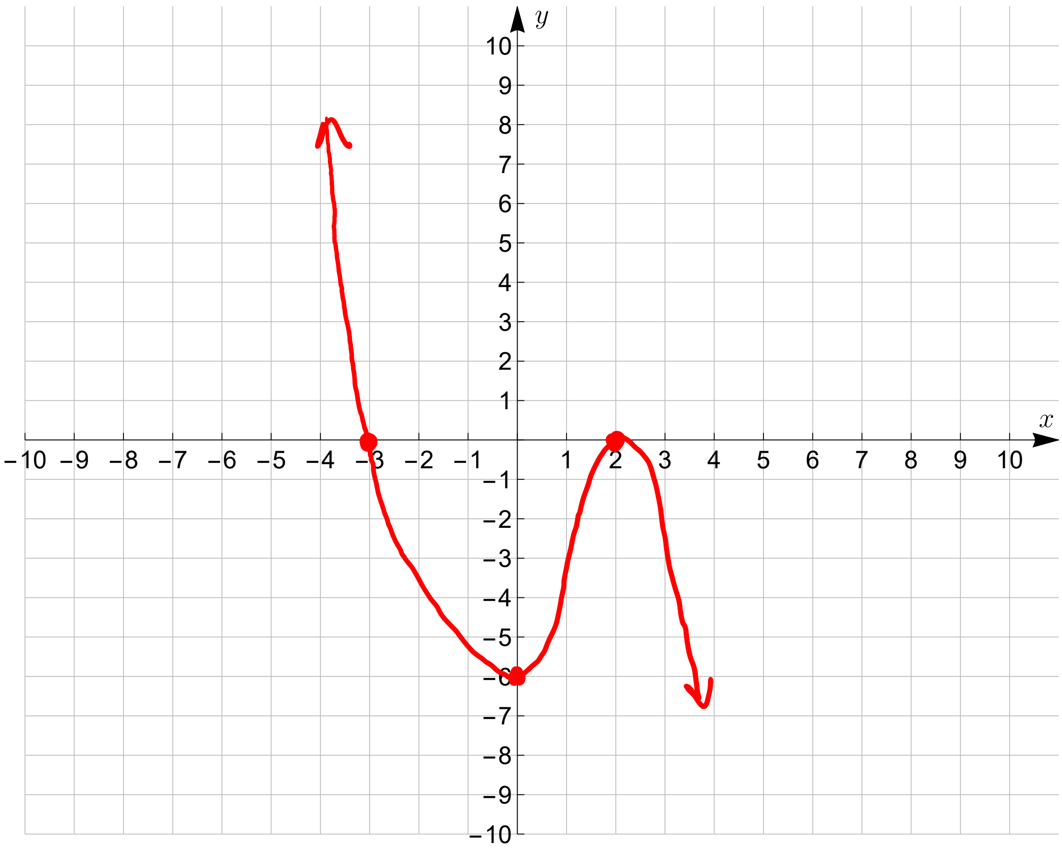


y-intercept: $f(0) = 2(0)^2 - 24(0) + 64 = 64$

off the graph

14. Graph the function. **Be sure to state the end behavior, x-intercepts, whether each x-intercept crosses the x-axis or touches the x-axis and turns around, and the y-intercept. Explain how you determined these.** Note: I've provided the factored form of the function so that you don't have to worry about factoring it yourself. (8 points each)

$$f(x) = -\frac{1}{2}x^3 + \frac{1}{2}x^2 + 4x - 6 = -\frac{1}{2}(x-2)^2(x+3)$$



end behavior: $-\frac{1}{2}x^3 \Rightarrow$ rises on left & drops on right

x-intercepts: $x=2$, $x=-3$

touches x-axis & turns around since exponent is 2

crosses through x-axis since exponent is 3

y-intercept: $f(0) = -\frac{1}{2}(0)^3 + \frac{1}{2}(0)^2 + 4(0) - 6 = -6$