Math 124E - Summer 2020 - Exam #3
DUE MONDAY, JULY 20, 2020 BY 11:59PM

Name: Answer Key

HONOR CODE: On my honor, I have neither given nor received any aid on this examination that is not explicitly allowed in the instructions.

Signature: ______________________________

Instructions: You may review the videos in MyMathLab or the course website, use the e-book, the MyMathLab homework and quizzes, and your calculator when working on this exam. **You may not receive help from anyone else, give help to anyone else, discuss any aspect of the exam or any items related to the exam with anyone else, or use any resources not specified in the previous sentence.** You may submit your answers and scratch work either on a printed copy of this exam or on your own paper. If you use your own paper, you do **not** need to copy the question; just be sure you clearly label which question the scratch work and answer belong to. If I can't tell with certainty which question any scratch work or answer belongs to, you will not receive credit for that work or answer. If you use your own paper, you **DO** need to copy the honor code above and sign it. To submit your scratch work and answers, you can either scan your work (if you have access to a scanner) or take pictures with your cell phone, then email me your scans or pictures. Be sure the writing in your scans or pictures is dark enough and clear enough that I can easily read what you've written. If I can't read what you've written, I can't give you credit for it. Make sure your final answers are clearly labeled. **SHOW ALL WORK ON THIS EXAM IN ORDER TO RECEIVE FULL CREDIT!!!**

No.	Score
1	/3
2	/6
3	/8
4	/4
5	/6
6	/8
7	/13
8	/6

No.	Score
9	/8
10	/8
11	/8
12	/4
13	/10
14	/8
Total	/100

1. Approximate each of the following using a calculator. Round your answers to four decimal places. (1 point each)

   (a) $3^{2.5}$

   15.5885

   (b) $e^{-1.8}$

   0.1653

   (c) $\ln 0.44$

   -0.8210

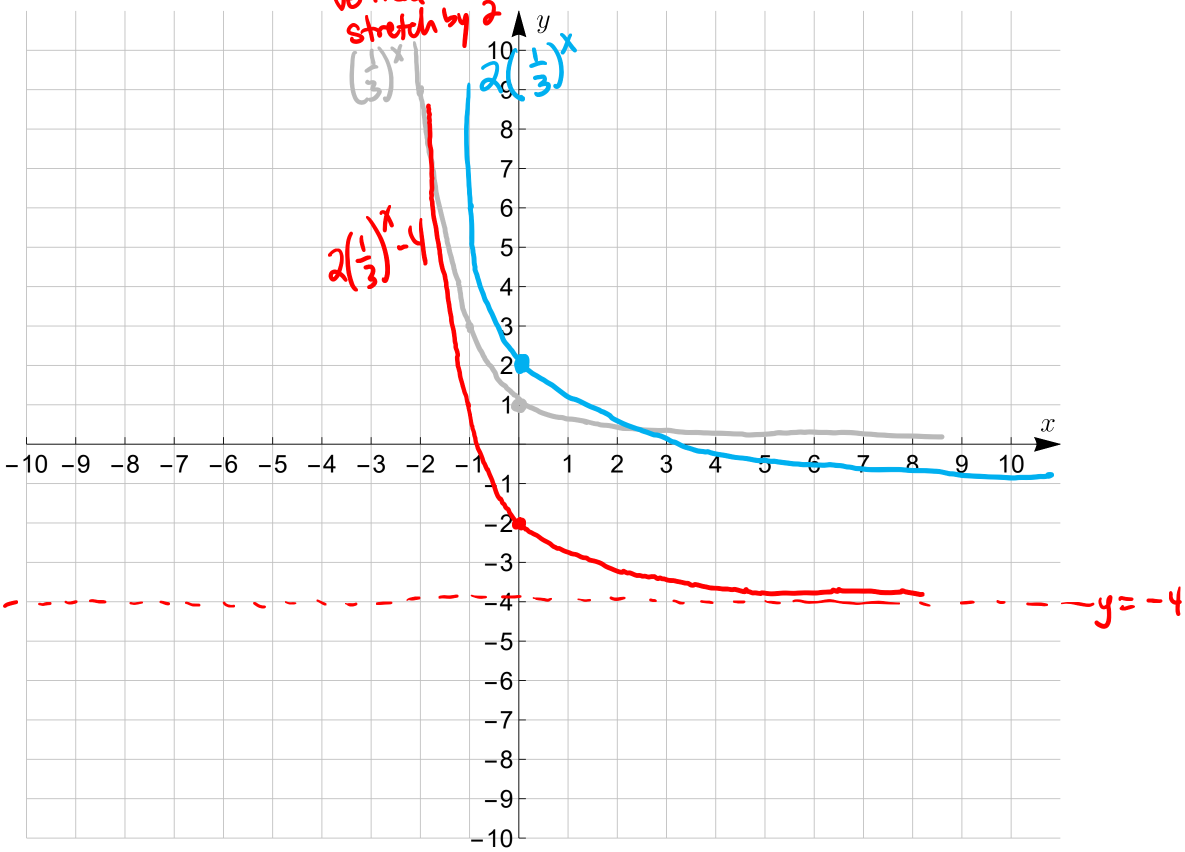
2. Use transformations of the graph of $f(x) = \left(\frac{1}{3}\right)^x$ to graph the given function. (6 points)

exponential decay

$$f(x) = 2\left(\frac{1}{3}\right)^x - 4$$

1st vertical stretch by 2

2nd down 4



3. Use the compound interest formulas $A = P\left(1 + \frac{r}{n}\right)^{nt}$ and $A = Pe^{rt}$ to find the accumulated value of an investment of \$12,000 for 5 years at an interest rate of 5% if the money is invested as follows. Round answers to the nearest cent. (4 points each)

(a) What is the accumulated value if the money is compounded quarterly?

Quarterly $\Rightarrow n = 4$

$A = 12{,}000\left(1 + \frac{0.05}{4}\right)^{4\cdot 5} = \$15{,}384.45$

(b) What is the accumulated value if the money is compounded continuously?

$A = 12{,}000e^{0.05(5)} = \$15{,}408.31$

4. (a) Write the equation in its equivalent logarithmic form. (2 points)

$$5^z = 4.1$$

$\log_5 4.1 = z$

(b) Write the equation in its equivalent exponential form. (2 points)

$$\log_t 6 = 2.85$$

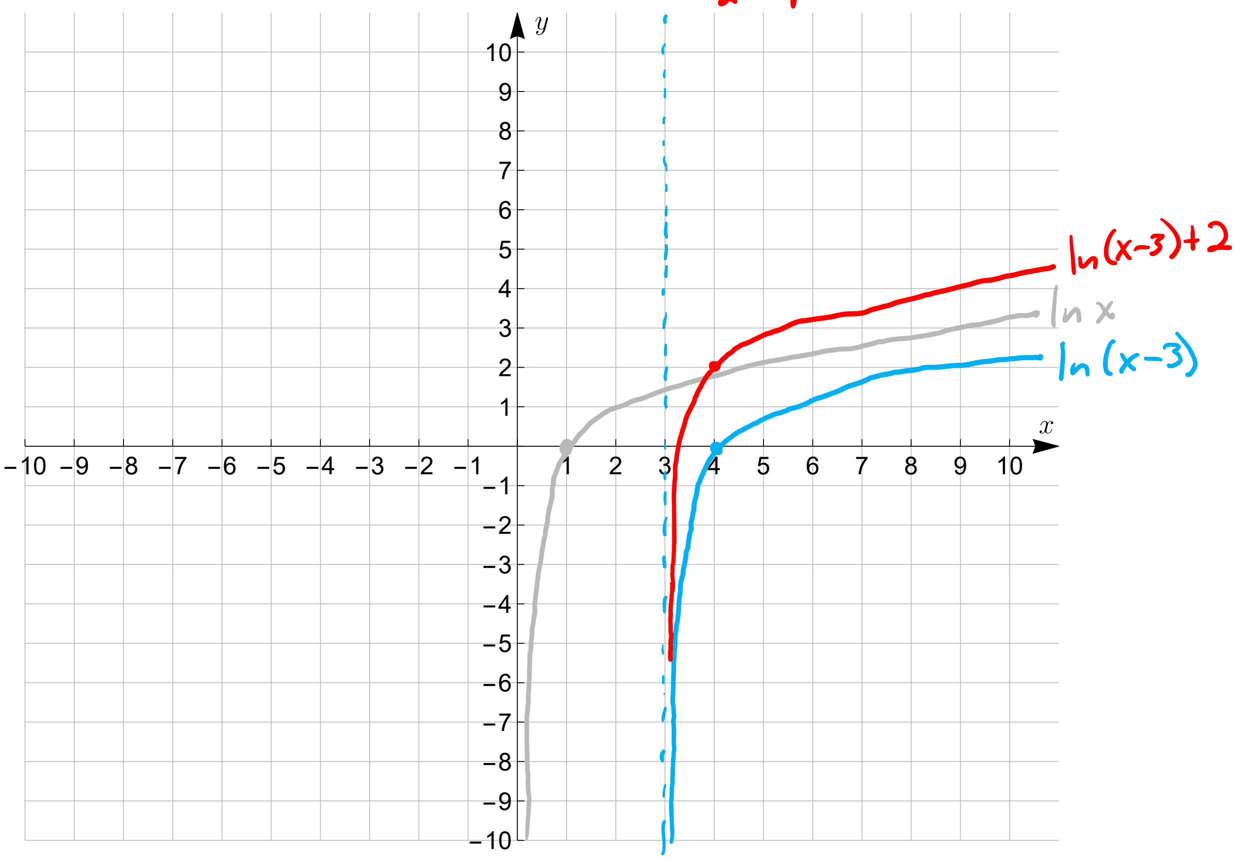
$t^{2.85} = 6$

5. Use transformations of the graph of $f(x) = \ln x$ to graph the given function. (6 points)

1st right 3

$$f(x) = \ln(x-3) + 2$$

2nd up 2



6. (a) Write as a sum/difference of logarithms. Express exponents as factors. Simplify as much as possible. (4 points)

$$\log_7\left(\frac{(x-3)^3}{2x+5}\right)$$

$$\log_7 (x-3)^3 - \log_7 (2x+5)$$

$$3\log_7 (x-3) - \log_7 (2x+5)$$

(b) Write as a single logarithm. Simplify as much as possible. (4 points)

$$4\ln(3x) + \ln(5y) - 3\ln(6x)$$

$$\ln (3x)^4 + \ln(5y) - \ln(6x)^3$$

$$\ln\left(\frac{(3x)^4(5y)}{(6x)^3}\right)$$

$$\ln\left(\frac{3^4 \overset{1}{\cancel{x^4}} \cdot 5y}{6^3 \cancel{x^3}}\right)$$

$$\ln\left(\frac{\cancel{3}\cdot\cancel{3}\cdot\cancel{3}\cdot 3\, x\cdot 5y}{\underset{2}{\cancel{6}}\cdot\underset{2}{\cancel{6}}\cdot\underset{2}{\cancel{6}}}\right)$$

$$\ln\left(\frac{15xy}{8}\right)$$

7. (a) Solve the following equation. Provide the exact value. (4 points)

$$16^{x+5} = 64^x$$

$$(4^2)^{x+5} = (4^3)^x$$

$$\cancel{4}^{2(x+5)} = \cancel{4}^{3x}$$

same base

$$2(x+5) = 3x$$

$$2\cancel{x} + 10 = 3x$$

$$-\cancel{2x} \qquad -2x$$

$$10 = x$$

(b) Solve the following equation. Provide the exact value and a decimal approximation rounded to two decimal places. (5 points)

$$10^{5x-1} = 20$$

$$\cancel{\log}\, \cancel{10}^{5x-1} = \log 20$$

$$5x - \cancel{1} = \log 20$$

$$\cancel{+1} \qquad +1$$

$$\frac{\cancel{5}x}{\cancel{5}} = \frac{1 + \log 20}{5}$$

$$x = \frac{1 + \log 20}{5} = 0.46$$

(c) Solve the following equation. Provide the exact value. (4 points)

$$\cancel{4}\log_4(3x-5) = 8$$

$$\frac{\quad}{\cancel{4}} \qquad \frac{\quad}{4}$$

Domain: $3x - \cancel{5} > 0$

$\quad +\cancel{5} \quad +5$

$\frac{\cancel{3}x}{\cancel{3}} > \frac{5}{3}$

$x > \frac{5}{3}$

$\log_4(3x-5) = 2$

$\cancel{4}^{\cancel{\log_4}(3x-5)} = 4^2$

$3x - \cancel{5} = 16$

$\quad +\cancel{5} \quad +5$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{21}{3}$

$x = 7$

8. The number of milligrams of a medication in a patient's body is decreasing exponentially $\left(A = A_0 e^{kt}\right)$. The patient is given 50 milligrams of the medication and after 6 hours, the patient still has 17 milligrams left in their body. It's not safe for the patient to take more medication until there is less than 5 milligrams of medication left in their body. What is the minimum amount of time after the patient's first dose of the medication they have to wait before they can safely take their next dose of the medication? Round your answer to the nearest hour. (6 points)

$$\frac{17}{50} = \frac{\cancel{50} e^{k(6)}}{\cancel{50}}$$

$$\frac{17}{50} = e^{6k} \Rightarrow \ln \frac{17}{50} = \cancel{\ln} \cancel{e}^{6k}$$

$$\frac{\ln \frac{17}{50}}{6} = \frac{\cancel{6}k}{\cancel{6}}$$

$$k = \frac{\ln \frac{17}{50}}{6} = -0.1798$$

$$\frac{5}{50} = \frac{\cancel{50} e^{-0.1798t}}{\cancel{50}}$$

$$.1 = e^{-0.1798t} \Rightarrow \ln .1 = \cancel{\ln} \cancel{e}^{-0.1798t}$$

$$\frac{\ln .1}{-0.1798} = \frac{\cancel{-0.1798}t}{\cancel{-0.1798}}$$

$t = 12.8 \Rightarrow$ minimum time = 13 hours

9. Solve the following system of equations by either the method of addition or the method of substitution. (8 points)

$$\begin{aligned} ① \quad x - y + 3z &= 8 \\ ② \quad 3x + y - 2z &= -2 \\ ③ \quad 2x + 4y + z &= 0 \end{aligned}$$

$① + ② \Rightarrow 4x + z = 6 \xrightarrow{\times(-13)} -52x - \cancel{13z} = -78$

$4 \times ① + ③ \Rightarrow 6x + 13z = 32 \longrightarrow 6x + \cancel{13z} = 32$

$\frac{\cancel{-46}x}{\cancel{-46}} = \frac{-46}{-46}$

$x = 1$

$4(1) + z = 6$

$\cancel{4} + z = 6$

$\cancel{-4} \qquad -4$

$z = 2$

③ $2x + 4y + z = 0$

$2(1) + 4y + 2 = 0$

$2 + 4y + 2 = 0$

$\cancel{4} + 4y = 0$

$\cancel{-4} \qquad -4$

$\frac{\cancel{4}y}{\cancel{4}} = \frac{-4}{4}$

$y = -1$

$(1, -1, 2)$

10. Graph the solution set of the following system of inequalities. (8 points)

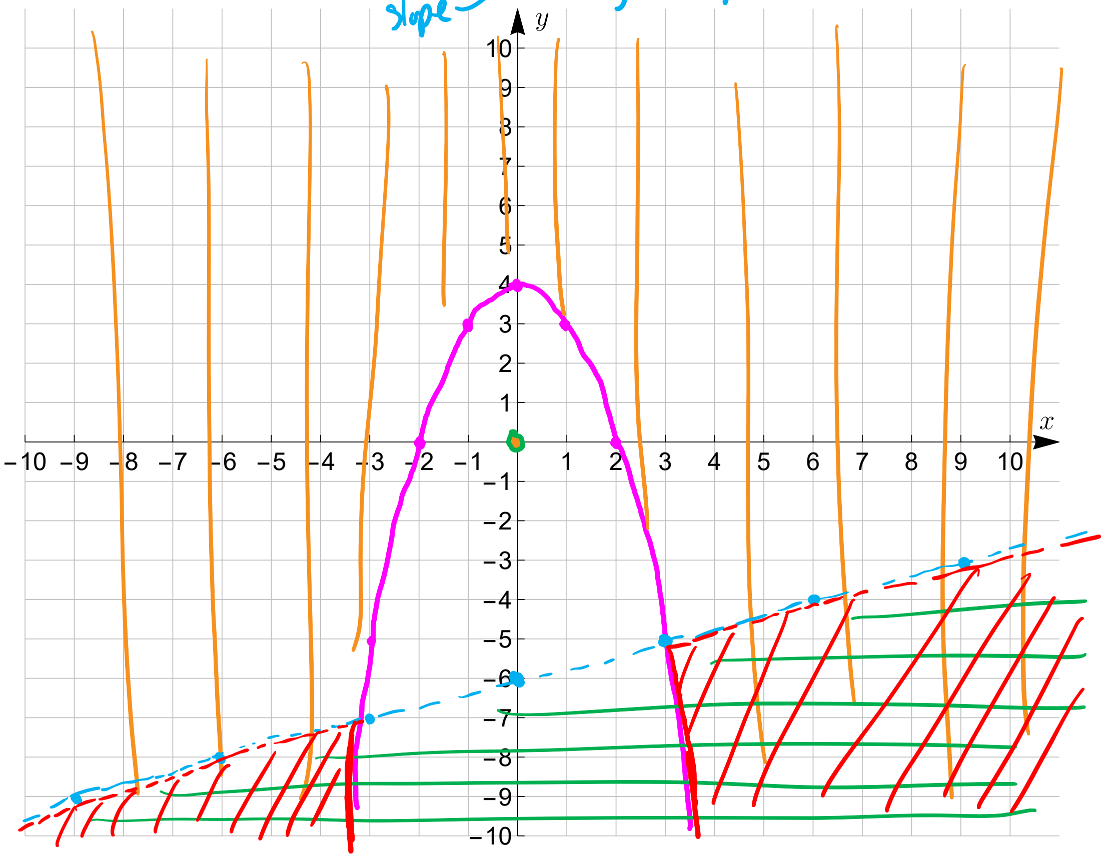
vertical reflection $\quad$ up 4

$$y \geq -x^2 + 4$$ Pick (0,0) : $0 \geq 0+4$ False

$$y < \frac{1}{3}x - 6$$ Pick: (0,0): $0 < 0-6$ False

y-intercept

slope



11. Write the first four terms of each of the following sequences. (4 points each)

(a) The sequence whose general term is given by the following.

$$a_n = \frac{2^n}{n^2+1}$$

$a_1 = \frac{2^1}{1^2+1} = \frac{2}{1+1} = \frac{2}{2} = 1$

$a_2 = \frac{2^2}{2^2+1} = \frac{4}{4+1} = \frac{4}{5}$

$a_3 = \frac{2^3}{3^2+1} = \frac{8}{9+1} = \frac{8}{10} = \frac{4}{5}$

$a_4 = \frac{2^4}{4^2+1} = \frac{16}{16+1} = \frac{16}{17}$

(b) The sequence defined by the following recursion formula.

$$a_1 = 5 \text{ and } a_n = 2a_{n-1} + 4 \text{ for } n \geq 2$$

$a_2 = 2a_1 + 4 = 2(5) + 4 = 10 + 4 = 14$

$a_3 = 2a_2 + 4 = 2(14) + 4 = 28 + 4 = 32$

$a_4 = 2a_3 + 4 = 2(32) + 4 = 64 + 4 = 68$

12. Perform the indicated sum. (4 points)

$$\sum_{j=1}^{3} \left(j^2 + j + 1\right)$$

$(1^2+1+1) + (2^2+2+1) + (3^2+3+1)$

$(1+1+1) + (4+2+1) + (9+3+1)$

$3 + 7 + 13$

23

13. (a) Use the appropriate formula for the general term (the n^{th} term) of a sequence to find a_{85} for the sequence with first term $a_1 = 15$ and common difference $d = -2$. (4 points)

Arithmetic $\Rightarrow a_n = a_1 + (n-1)d$

$$a_{85} = a_1 + (85-1)d$$

$$a_{85} = 15 + 84(-2) = -153$$

(b) Write a formula for the n^{th} term of the following sequence, then use that formula to find a_9. (6 points) $729, 243, 81, 27, \ldots$

$a_1 = 729$ $\times \frac{1}{3}$

$a_2 = 243$ $\times \frac{1}{3}$

$a_3 = 81$ $\times \frac{1}{3}$

$a_4 = 27$

Geometric & $r = \frac{1}{3} \Rightarrow a_n = a_1 r^{n-1}$

$$a_n = 729\left(\frac{1}{3}\right)^{n-1}$$

$$a_9 = 729\left(\frac{1}{3}\right)^{9-1} = 729\left(\frac{1}{3}\right)^8 = \frac{1}{9}$$

14. Use the appropriate sum formula to find the indicated sum. (4 points each)

(a) Find $1 \overset{\times 4}{\curvearrowright} 4 \overset{\times 4}{\curvearrowright} 16 + \cdots + 4096$, the sum of the first 7 powers of 4, starting with 4^0.

Geometric & $r=4 \Rightarrow S_n = \dfrac{a_1(1-r^n)}{1-r}$

$$S_7 = \frac{1(1-4^7)}{1-4} = \frac{1(1-16{,}384)}{-3}$$

$$= \frac{1(-16{,}383)}{-3}$$

$$= \frac{-16{,}383}{-3}$$

$$= 5461$$

(b) Find $5 \overset{+5}{\curvearrowright} 10 \overset{+5}{\curvearrowright} 15 \overset{+5}{\curvearrowright} 20+25+\cdots+250$, the sum of the first 50 positive, integer multiples of 5.

Arithmetic & $d=5 \Rightarrow S_n = \frac{n}{2}(a_1+a_n)$

$$S_{50} = \frac{50}{2}(5+250)$$

$$= 25(255)$$

$$= 6375$$